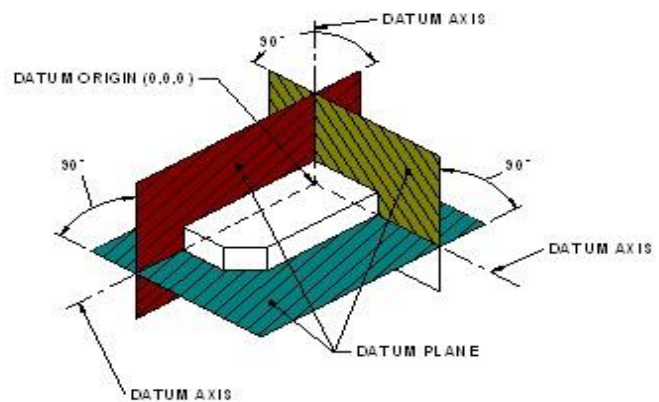# Formulas for Gear Calculation – External Gears

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Formulas for gear calculation – external gears

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The **involute gear** profile is the most commonly used system for gearing today. In an involute gear, the profiles of the teeth are *involutes of a circle.* (The involute of a circle is the spiraling curve traced by the end of an imaginary taut string unwinding itself from that stationary circle.)
In involute gear design, all contact between two gears occurs in the same fixed, flat plane even as their teeth mesh in and out. Further, the contacting surfaces are always perpendicular to the plane of contact, so the dominant contact forces (in a well lubricated system) are always parallel to the plane. This way, the moment arms are kept constant. This is key to minimizing the torque/speed variations which produce vibration and noise in lower quality gears. Note that the involute profile does not prevent the teeth from scraping each other every time they mesh, and this is the dominant source of wear. It is not possible to design a gear tooth profile which rolls through the mesh without friction.

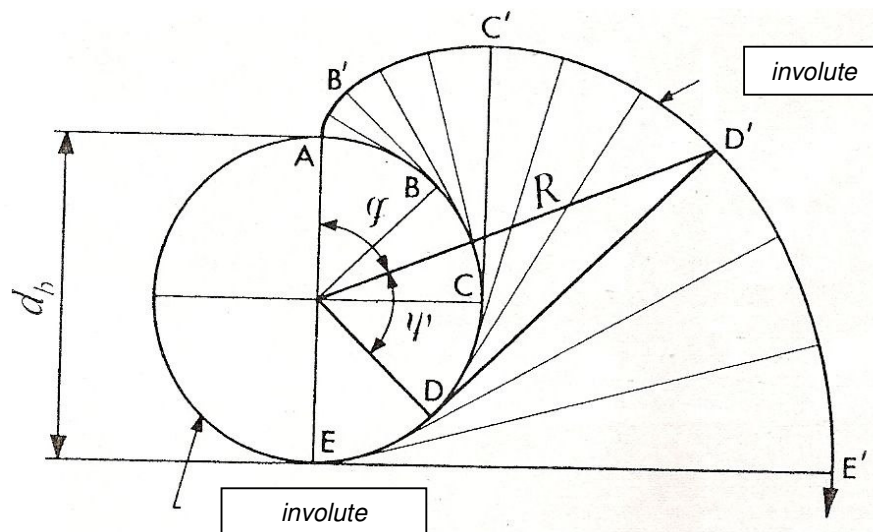The figure N°1 show the involute curve generation with the most important elements.

Fig. N°1

The parametric equations of involute of a circle are::

$$R = \frac{r_g}{\cos\psi} = r_g \cdot \sec\psi$$

$$\varphi = \tan\psi - \psi = inv\,(\psi)$$

Because we must use many formulas, it's better to show the list of symbols and indices utilized.

|	Meaning of symbols				
a	Center distance		m	Module	
α	Pressure angle		Q	Dimension over pins or balls	
β	Helix angle		r	Radius	
d	Diameter		R_a	Radius to start of active profile	
g	Length of contact		s	Tooth thickness on diameter d	
g_1	Legth of recession		$\bar{s}_{os}$	Chordal thickness	
g_2	Length of approach		t	Pitch	
h_f	Dedendum		w	Chordal thickness over z' teeth (spur gears)	
h_k	Addendum		W	Chordal thickness over z' teeth (helical gears)	
h_0	Corrected addendum		z	Number of teeth	
h_r	Whole depth		x	Profile correction factor	
l	Tooth space				
	Meaning of indices				
b	Reffered to rolling diameter		n	Reffered to normal section	
c	Referred to roll diameter of basic rack		o	Reffered to pitch diameter	
f	Reffered to root diameter		q	Reffered to the diameter throug balls center	
g	Reffered to base diameter		r	Reffered to balls	
k	Referred to outside diameter		s	Reffered to transverse section	
i	Refferred to equivalent		w	Reffered to tool	

<u>*Determination of base tooth thickness from a known thickness and vice-versa.*</u>

$$s_g = (\psi + 2\vartheta)\cdot r_g \qquad \text{where} \quad \psi = \frac{s}{r}$$

and vice-versa is:

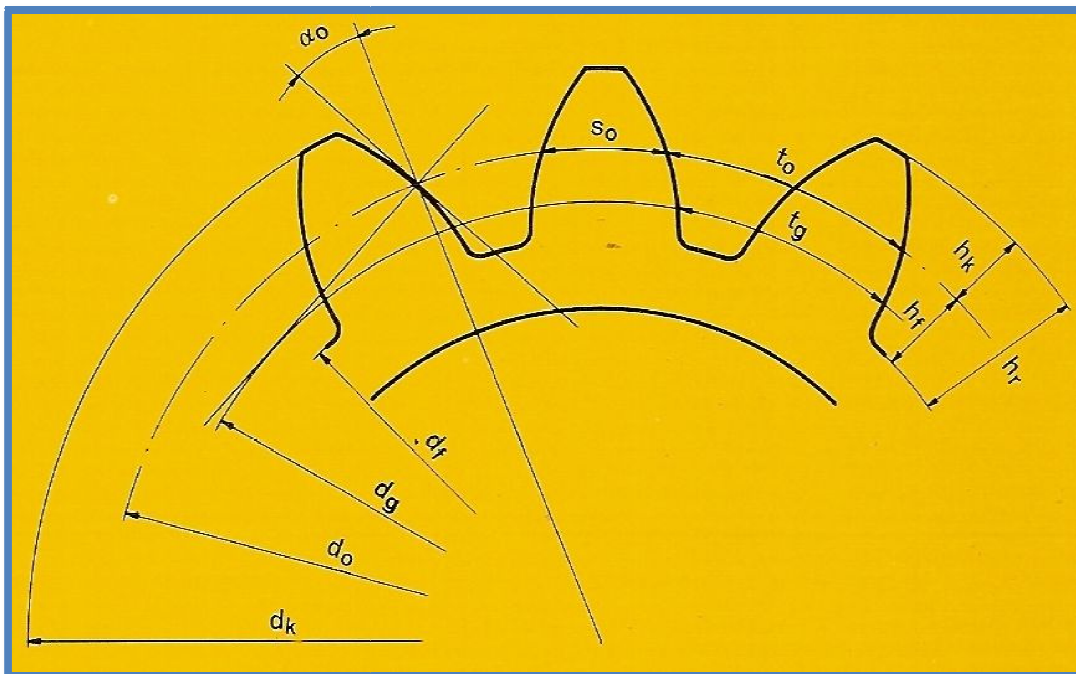$$s = \left(\frac{s_g}{r_g} - 2\cdot inv\,\alpha\right)\cdot r$$

Where we can see $\vartheta = inv\,\alpha$ where α is the pressure angle in the points E and D. We can understand immediately the importance of the function $inv\,\alpha$.
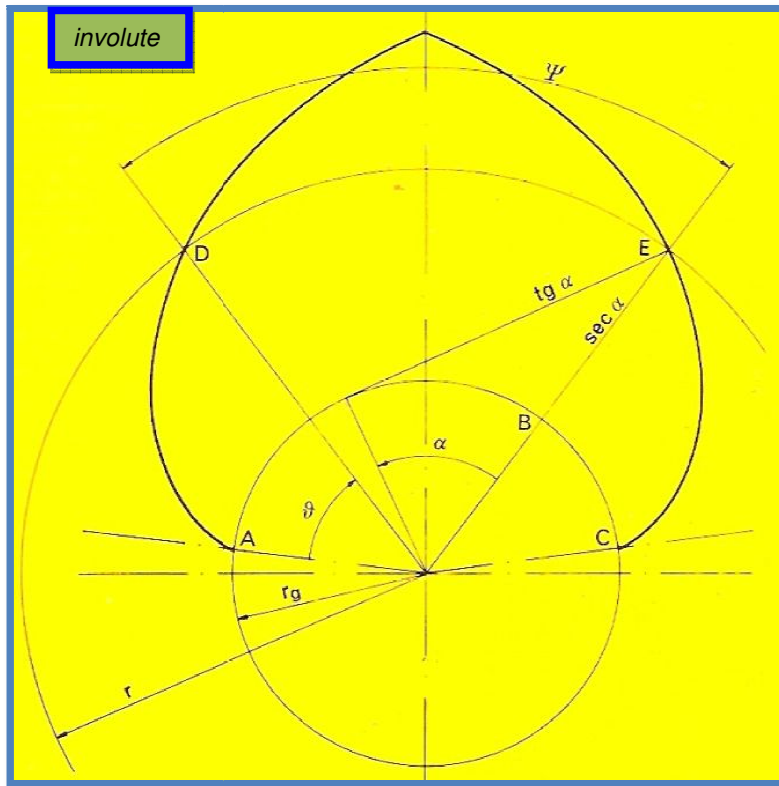
Fig. N°2

Cylindrical spur gears with standard profile

On base to figure N°3 the following relation are valid:

Fig. N°3

$$d_o = m \cdot z$$
$$t_o = m \cdot \pi$$
$$d_g = d_o \cdot \cos \alpha_o$$
$$t_g = t_o \cdot \cos \alpha_o$$
$$h_k = m$$
$$h_f = \frac{7}{6} \cdot m \quad \text{or} \quad h_f = \frac{5}{4} \cdot m$$

$$h_r = h_k + h_f$$
$$d_k = d_o + 2h_k$$
$$d_f = d_o + 2h_f$$
$$S_o = \frac{m \cdot \pi}{2}$$

Cylindrical spur gears with corrected profile
The distance between the pitch line of the rack (a) (see figure N°4) and the rolling line (b) is called corrected profile $\boldsymbol{x \cdot m}$. The corrected profile is positive when the pitch line of the rack is above the cutting pitch circle of the gear. In the opposite case there is minus correction.

a)- *Without center distance variation (fig.N°4):*
$$h_k = m + x \cdot m$$
$$h_f = \frac{7}{6} \cdot m - x \cdot m \quad \text{or} \quad h_f = \frac{5}{4} \cdot m - x \cdot m$$
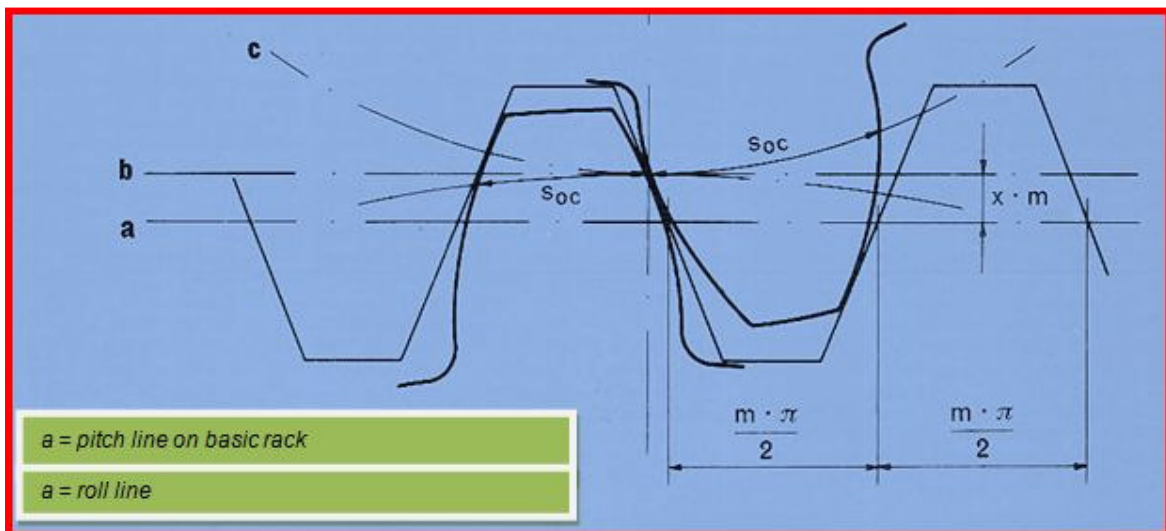$$S_{oc} = \frac{m \cdot \pi}{2} + 2 \cdot x \cdot m \cdot \tan \alpha_o$$

a = pitch line on basic rack
a = roll line

Fig.N°4

b)- *With center distance variation (fig.N°5)*

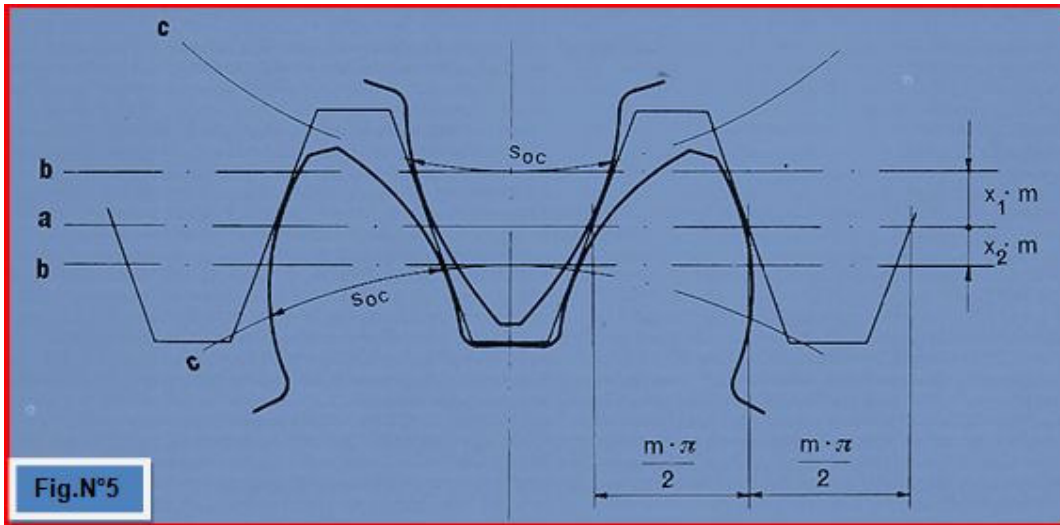$$inv\, \alpha_b = inv\, \alpha_o + 2 \tan \alpha_o \frac{x_1 + x_2}{z_1 + z_2}$$
$$a_b = a \frac{\cos \alpha_o}{\cos \alpha_b} \qquad d_b = \frac{d_g}{\cos \alpha_b}$$
$$s_b = r_b \left[\frac{S_{oc}}{r_o} + 2(inv\, \alpha_o - inv\, \alpha_b) \right]$$

$$k = \frac{z_1 + z_2}{2} \left[2 \frac{x_1 + x_2}{z_1 + z_2} - \left(\frac{\cos \alpha_o}{\cos \alpha_b} - 1 \right) \right]$$

$$d_k = m(z + 2 + 2x - 2k)$$

$$h_r = m(2{,}25 - k) \quad \text{or} \quad h_r = m(2{,}167 - k)$$

Fig.N°5

Cylindrical helical gears with standard profile (Fig. N°6)

$$d_o = m_s \cdot z \qquad\qquad t_{os} = m_s \cdot \pi$$
$$d_g = d_o \cdot \cos \alpha_{os} \qquad\qquad t_{gs} = t_{os} \cdot \cos \alpha_{os}$$
$$m_n = m_s \cdot \cos \beta_o \qquad\qquad \tan \alpha_{on} = \tan \alpha_{os} \cdot \cos \beta_o$$
$$t_{on} = m_n \cdot \pi \qquad\qquad t_{gn} = t_{on} \cdot \cos \alpha_{on}$$
$$h_k = m_n \qquad\qquad h_f = \frac{7}{6} \cdot m_n \quad \text{or} \quad h_f = \frac{5}{4} \cdot m_n$$
$$h_r = h_k + h_f \qquad\qquad d_k = d_o + 2h_k$$
$$d_f = d_o + 2 \cdot h_f \qquad S_{on} = \frac{\pi \cdot m_n}{2} \qquad S_{os} = \frac{\pi \cdot m_s}{2}$$

Fig. N°6
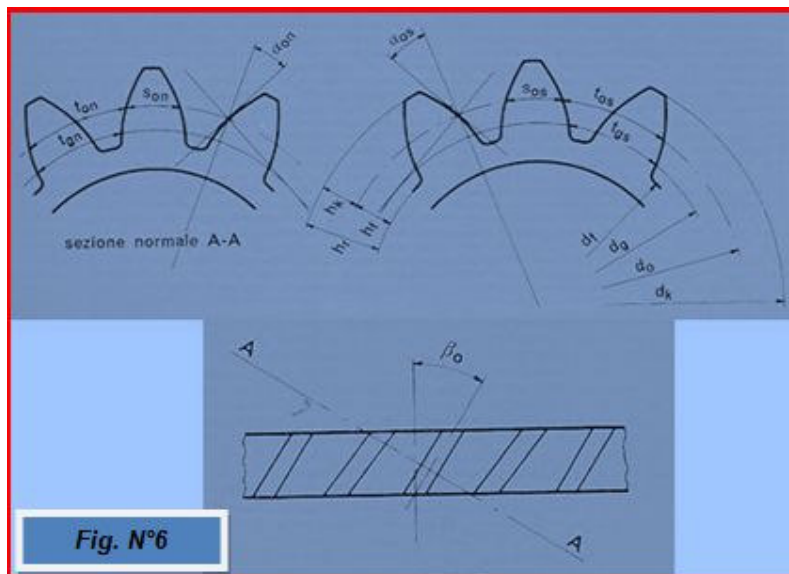
<u>*Cylindrical helical gears with corrected profile*</u>

a)- Without center distance variation (fig.N°4).

$$h_k = m_n + x \cdot m_n$$

$$h_f = \frac{7}{6} \cdot m_n - x \cdot m_n \quad \text{or} \quad h_f = \frac{5}{4} \cdot m_n - x \cdot m_n$$

$$s_{onc} = \frac{m_n \cdot \pi}{2} + 2 \cdot x \cdot m_n \cdot \tan \alpha_{on}$$

$$s_{osc} = \frac{m_s \cdot \pi}{2} + 2 \cdot x \cdot m_n \cdot \tan \alpha_{os}$$

b)- With center distance variation (fig. N°5)

$$inv\, \alpha_{bs} = inv\, \alpha_{os} + 2 \tan \alpha_{on} \frac{x_1 + x_2}{z_1 + z_2}$$

$$a_b = a \frac{\cos \alpha_{os}}{\cos \alpha_{bs}} \qquad\qquad d_b = \frac{d_g}{\cos \alpha_{bs}}$$

$$s_{bs} = r_b \left[\frac{s_{osc}}{r_o} + 2(inv\, \alpha_{os} - inv\, \alpha_{bs}) \right]$$

$$k = \frac{z_1 + z_2}{2} \left[2 \frac{x_1 + x_2}{z_1 + z_2} - \frac{1}{\cos \beta_o} \left(\frac{\cos \alpha_{os}}{\cos \alpha_{bs}} - 1 \right) \right]$$

$$d_k = m_n \left(\frac{z}{\cos \beta_o} + 2 + 2x - 2k \right)$$

$$h_r = m_n(2{,}25 - k) \quad \text{or} \quad h_r = m_n(2{,}167 - k)$$

<u>*Length of contact and contact radius R_a*</u>

$$\rho_{k1} = \sqrt{r_{k1}^2 - r_{g1}^2} \qquad\qquad \rho_{k2} = \sqrt{r_{k2}^2 - r_{g2}^2}$$

$$g = \rho_{k1} + \rho_{k1} - a_b \cdot \sin \alpha_b$$

$$g_1 = \rho_{k1} - r_{b1} \cdot \sin \alpha_b \qquad\qquad g_2 = \rho_{k2} - r_{b2} \cdot \sin \alpha_b$$

$$R_{a1o1} = \sqrt{(\rho_{k1o1} - g)^2 + r_{g1o2}^2}$$
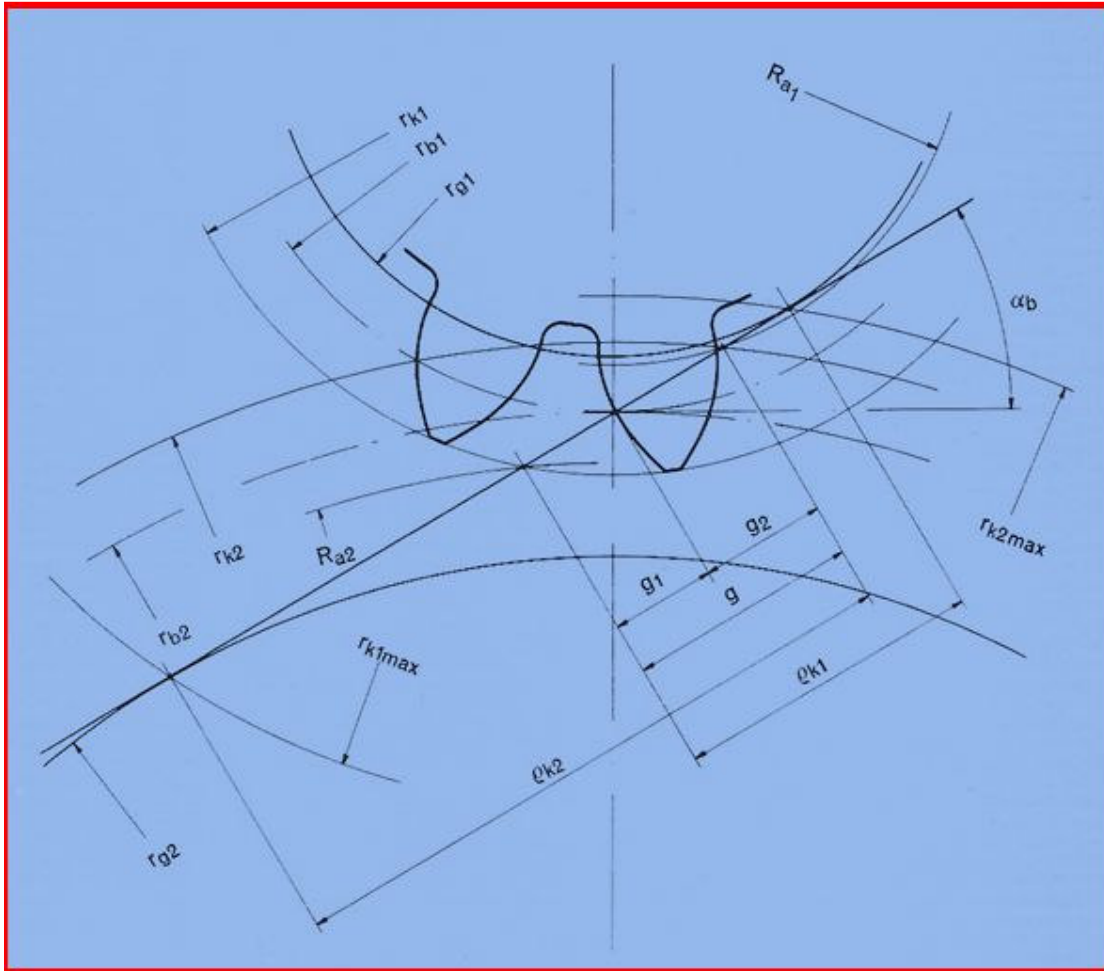
Fig. N°9

Interference
Maximum outside diameter without interference:

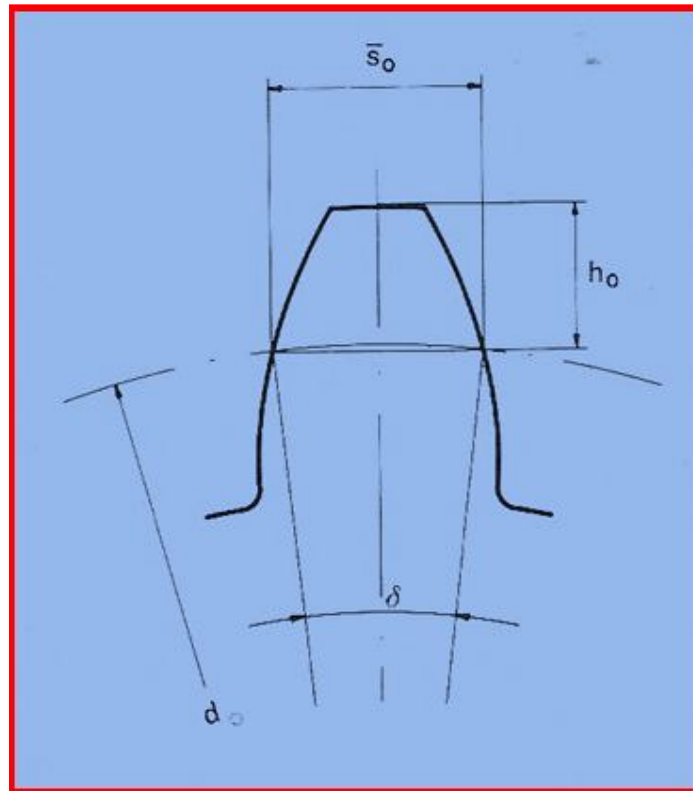$$d_{kmax} = \sqrt{d_g^2 + (2a_b \cdot \sin \alpha_b)^2}$$

Chordal thickness and corrected addendum (fig.10)

$$\frac{s_o}{\pi \cdot d} = \frac{\delta}{2 \cdot \pi} \quad \text{and} \quad \delta = \frac{2 \cdot s_o}{d}$$
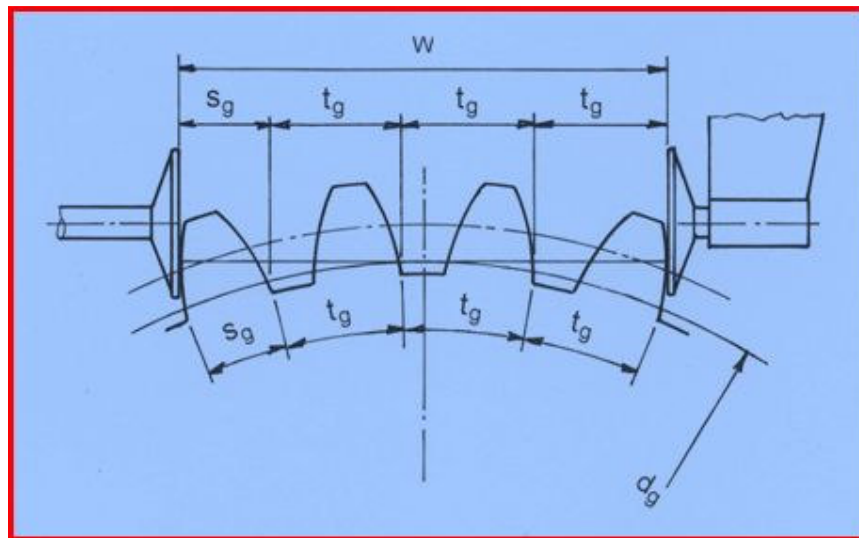
$$\bar{s}_o = d \cdot \sin \frac{\delta}{2}$$

$$h_o = h_k + \frac{d}{2}\left(1 - \cos \frac{\delta}{2}\right)$$

Fig. N°10

Span measurement over Z' teeth

Fig. N°11

For spur gears:

$$w = m \cdot \cos \alpha_o \cdot \left[(z' - 1)\pi + \frac{s_o}{m} + z \cdot inv \, \alpha_o \right] + 2 \cdot x \cdot m \cdot \sin \alpha_o$$

The number of teeth Z in the span is:

$$z' = z\frac{\alpha_o}{180°} + 0{,}5 \qquad \text{(with } \alpha_o \text{ in degree)}$$

The result should be rounded off to the next highest whole number.
For gear with profile correction $\geq 0{,}4 \cdot m$ teh formula is:

$$z' = z\frac{\alpha_o}{180°} + 0{,}5 + z \cdot \frac{\tan \alpha_x}{\pi} - (z + 2x)\frac{\tan \alpha_o}{\pi} \qquad \text{where}$$

$$\tan \alpha_x = \sqrt{\tan^2 \alpha_o + 4\frac{x}{z}\sec^2 \alpha_o + 4\left(\frac{x}{z}\right)^2 \sec^2 \alpha_o}$$

For helical gears:

$$w = m_n \cdot \cos \alpha_{on} \cdot \left[(z' - 1)\pi + \frac{S_{on}}{m_n} + z \cdot inv\, \alpha_{os}\right] + 2 \cdot x \cdot m_n \cdot \sin \alpha_{on}$$
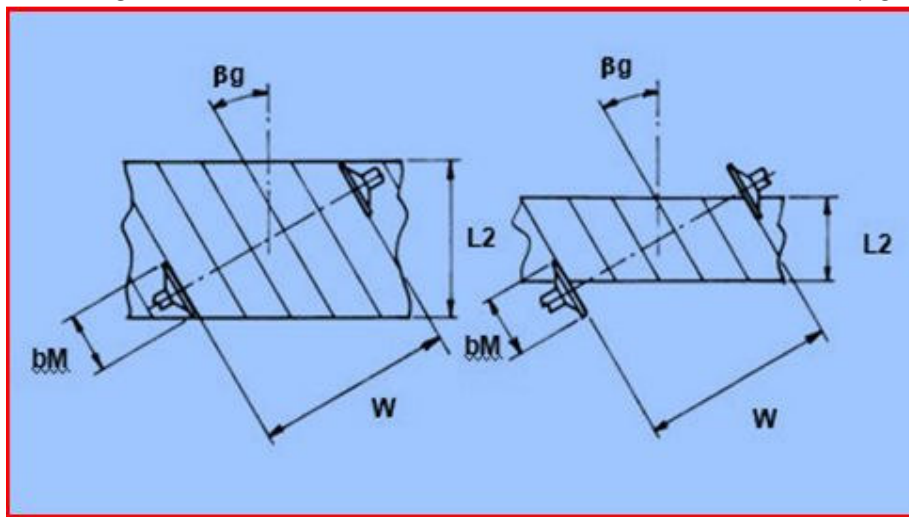
where

$$z' = z\left(\frac{\alpha_{os}}{180°} + \frac{\tan \alpha_{os} \cdot \tan^2 \beta_g}{\pi}\right) + 0{,}5 \qquad \text{(con } \alpha_o \text{ in degree)}$$

This method of checking can only be used when

$$b \geq w \cdot \sin \beta_g + b_M \cdot \cos \beta_g$$

where β_g is the length of the contact lines between little dishes and teeth (figura N°12).

Fig.N°12

If the gear has a large profile correction, then z should be calculated as follow:

$$z' = z\frac{\alpha_{os}}{\pi} + 0{,}5 + \frac{z}{\pi} \cdot \frac{\sqrt{\tan^2 \alpha_{on} + 4\frac{x}{z}\cos \beta_o \left(1 + \frac{x}{z}\cos \beta_o\right)(\tan^2 \alpha_{on} + \cos^2 \beta_o)}}{\cos \beta_o(\sin^2 \alpha_{on} + \cos^2 \beta_o \cdot \cos^2 \alpha_{on})} -$$

$$-\frac{z}{\pi}\tan \alpha_{os} - 2\frac{x}{\pi}\tan \alpha_{on}$$
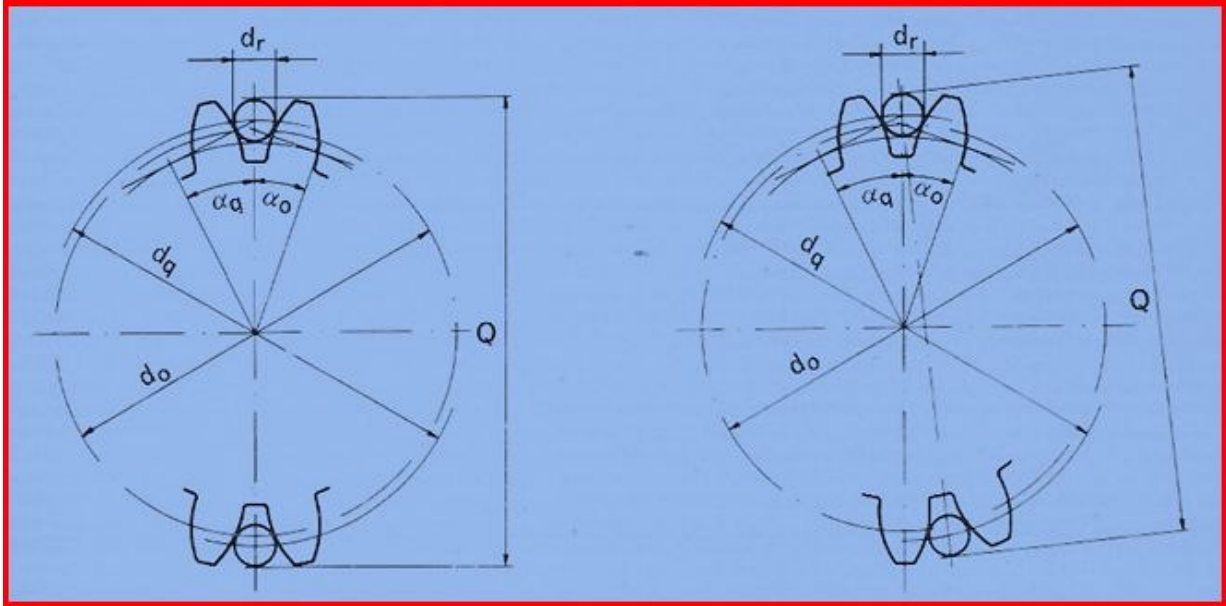
Dimension over pins or balls

Fig.N°13

Spur gears with even number of teeth

$$inv\ \alpha_q = inv\ \alpha_o + \frac{d_r}{2r_o \cdot \cos \alpha_o} - \frac{l_o}{2r_o} \qquad \text{from which we have} \quad \alpha_q$$

$$r_q = r_o \frac{\cos \alpha_o}{\cos \alpha_q} \qquad\qquad Q = 2 \cdot r_q + d_r$$

Spur gears with odd number of teeth

With the same α_q e r_q we obtain $\qquad Q = 2 \cdot r_q \cdot \cos\frac{\pi}{2z} + d_r$

Helical gear with even number of teeth

$$inv\ \alpha_{qs} = inv\ \alpha_{os} + \frac{d_r}{2r_{os} \cdot \cos \beta_o \cos \alpha_{on}} - \frac{l_{os}}{2r_{os}}$$

$$r_{qs} = r_{os} \frac{\cos \alpha_{os}}{\cos \alpha_{qs}} \qquad\qquad Q = 2 \cdot r_{qs} + d_r$$

Helica gears with odd number of teeth

With the same α_{qs} e r_{qs} we obtain $\qquad Q = 2 \cdot r_{qs} \cdot \cos\frac{\pi}{2z} + d_r$